Rasch, Frisch, and Two Fishers:

A Social History of the Econometric Origins of Some Widely Used Psychometric Models

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Abstract

In 1979, the Danish mathematician Georg Rasch recounted a 1959 visit with his former teacher, and later economics Nobel Prize winner, Ragnar Frisch. Frisch prompted Rasch to formalize his work in a separability theorem. Previously unnoted is that Frisch's close colleague, Irving Fisher, had previously formulated an econometric "separation theorem," for which Frisch defined necessary and sufficient conditions of its satisfaction. Rasch attributes his model to Ronald Fisher's concept of statistical sufficiency. Recent renewed appreciation for Irving Fisher's contributions to the foundations of neoclassical economics suggests that Rasch's models may have more significance for econometrics than has yet been accorded.

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On the Demand for Improved Measurement

Measurement models based in Rasch's (1960, 1961, 1965, 1977) parameter separability theorem are widely used in education, psychology, health care, economics, and the social sciences generally (Andrich, 1988; Bond and Fox, 2007; Fischer and Molenaar, 1995; Fisher and Wright, 1994), though their fundamental integration of qualitative meaning and quantitative rigor (Fisher, 2004b) remains an elusive concept to most researchers. In addition, it has repeatedly been shown that Rasch models stand to improve measurement quality not only in the obvious domains involving tests, surveys, performance assessments, etc., but in other areas, such as genomic medicine (Li and Hong, 2001; Markward, 2004) or medical diagnostic tests (Cipriani, Fox, Kuhder, et al., 2005), typically taken for granted as having proper measurement in place.

Another instance of this phenomenon of unrecognized potential for marked improvement is in the domain of the measurement of capital. Obviously, the quantification of human, social, and natural capital requires new and innovative measurement approaches (Andriessen, 2003; Fisher, 2002, 2004a, 2004c, 2008; Gautschi, 2001), but our longstanding measures of manufactured and liquid capital are as solid as can be, aren't they? There aren't really any major areas of significant concern in traditional capital accounting practices, are there?

Don't count on it! Even before the implosions of Enron, Tyko, and Worldcom, there were heated debates on how to best estimate the aggregate value of capital (Brown, 1980; Denny, 1980; Diewert, 1980; Putnam & Goss, 2002, p. 8). Since these debacles, however, the "perils of numeracy"—that is, the dangers of letting numbers think for us—are far more widely recognized (Bogle, 2003). And these events have also brought relevance to previous, independently produced work in the philosophy and history of science as to the way the meanings of numbers are contingent on a wide array of qualitative social, linguistic, technological, and cultural factors (for instance, Fisher, 2003a; Latour, 1987, pp. 247-57; Porter, 1999).

Opportunities for improved measurement are legion, especially in building those qualitative bridges to reality that were called for by Irving Fisher in his original work on the construction of aggregate capital indices (I. Fisher, 1967/1922, p. 274; see Boumans, 2001a, p. 340). Though few psychometric theoreticians, researchers or analysts think of their work as belonging to the general domain of economics and econometrics, improving the management of human, social, and natural capital requires just that expertise in qualitatively meaningful quantification that Rasch measurement can bring to bear.

And there is considerable evidence that Rasch's involvement in economic research activities with a number of leading economists may have had decisive influence on his approach to measurement. Beyond that, might it be that many of the major methodological moves efficiently integrated into comprehensive applications of Rasch models are found to exist as separate and isolated procedures routinely employed in econometrics? If so, Rasch's unidimensional models for measurement might be a source for innovative ways of countering the perils of numeracy with more thoughtful emphases on the creation and management of locally meaningful and globally comparable measures.

Exploration of these issues begins by recounting various influences on Rasch and his work exerted directly by colleagues such as Ragnar Frisch and Ronald Fisher, and those exerted indirectly by the broader social and intellectual context. Particularly intriguing is the overlap of key concepts, such as parameter separation and statistical sufficiency, in the independent bodies of work produced by Rasch and Ronald Fisher, on the one hand, and by Frisch and Irving Fisher, on the other.

Frisch's Decisive and Direct Influence on Rasch

In a 1979 interview (Andrich, 1997, 2005), conducted a year before he died, Rasch recounted three important events in his life. First, in the Spring of 1935, he studied with Ragnar Frisch in Oslo, and, second, he went to London to study with Ronald Fisher for the 1935-6 academic year. In the interview, Rasch mentions Frisch's confluence analysis and his occasional use of it, but stressed repeatedly how important Fisher's concept of the sufficient statistic was to him in the development of his approach to measurement.

The third key event occurred later, in 1959, when Rasch visited Frisch and recounted some of his recent work for him, including the development of the model for which Rasch is now well-known. As Rasch (1977, p. 63; also see Andersen & Olsen, 2001; Andrich, 1997, 2005; Olsen, 2003; Wright, 1980, pp. xvi-xviii) remembered it, his

> understanding of what the model entails tarried several years. At the 1959 anniversary of the University of Copenhagen the highly esteemed Norwegian economist Ragnar Frisch - later Nobel Prize winner - was to receive an honorary doctorate. I visited him by appointment the next day, and when our business was finished he asked me what I had been doing in the 25 years since I stayed at his institute in Oslo for a couple of months to study a new technique of statistical analysis [confluence analysis] that he had developed.

Rasch then proceeds to recount for Frisch his development of a model for measuring students' reading speeds, writing out a series of equations (II:1 - II:7 in Rasch, 1977):

> On seeing (II:7) Frisch opened his eyes widely and exclaimed: 'It (the person parameter) was eliminated, that is most interesting!' And this he repeated several times during our further conversation. To which I of course agreed every time - while I continued reporting the main results of the investigation and some of my other work.
>
> Only some days later I all of a sudden realized what in my exposition had caused this reaction from Ragnar Frisch. And immediately I saw the importance of finding an answer to the following question: 'Which class of probability models has the property in

> common with the Multiplicative Poisson Model, that one set of parameters can be eliminated by means of conditional probabilities while attention is concentrated on the other set, and vice versa?'
>
> What Frisch's astonishment had done was to point out to me that the possibility of separating two sets of parameters must be a fundamental property of a very important class of models.

Rasch only then, in light of the insight prompted by Frisch's astonishment, generalized his results into a formal model supported by what he called a separability theorem and the concept of specific objectivity (Rasch, 1960, 1961, 1966a, 1966b, 1977). Rasch wasted no time in contacting L. J. Savage at the University of Chicago about a visiting appointment. Savage asked Ben Wright to act as Rasch's host, and Wright came to be profoundly influenced by what Rasch had to say, eventually making several trips to Denmark to study with Rasch (Andrich, 1995; Wright, 1988).

Frisch's Decisive and Indirect Social Influence on Rasch

But why did Rasch contact Savage in Chicago? How did it happen that Savage was inclined to extend an invitation to him? What prior history did they share that would have given Rasch such credibility with Savage? Wright (1988), Andrich's (1979) interview with Rasch, and the Cowles Commission Report for 1947 (Cowles, 1947; Linacre, 1998) provide some answers, as they document Rasch's social connections and the role Rasch played in the Commission's investigations into economic problems.

Rasch is identified twice in the 1947 report as a Fellow of the University of Chicago. He gave two papers late in the year that served as the starting points for staff discussions. One was entitled, "A Biometric Multidimensional Model", and the other, "Remarks on Estimation."

On November 20, 1947, Rasch gave a seminar in Chicago under the heading, "Statistical Analysis of Growth Curves." His was the last of the year. It was preceded by seminars given by later Nobel Laureates Herbert Simon (1978), Milton Friedman (1976), and Ragnar Frisch (1969).

Others giving seminars in the same series that year included Rensis Likert, Sewall Wright, and Harold Hotelling. The papers given by Friedman and Rasch were presented at joint meetings of the Cowles Commission and the Statistical Techniques Group of the Chicago Chapter of the American Statistical Association, which had Kenneth Arrow, a 1972 Nobelist, as its chairman. L. J. Savage gave a paper at the Cowles Commission in 1947, as well. He spoke about estimating parameters for a continuous stochastic process, a topic closely related to the work Rasch would come to present at Chicago in 1960.

This would seem, then, to be when Rasch and Savage came to be acquainted. But how did Rasch come to be at the Cowles Commission in the first place? It would seem natural that his presence there would have come about through his association with Frisch. And indeed, as is summarized by Wright (1980, pp. xii-xiii) from Andrich's (1979) interview with Rasch,

> …a friendship with Chester Bliss made in London in 1935 brought Rasch to the United States in 1947 to participate in the founding of the Biometrics Society (Rasch 1947) and in the postwar reorganization of the International Statistical Institute in Washington. In the course of these meetings Tjalling Koopmans, a fellow student of Ragnar Frisch's confluence analysis and Fisher's sufficient statistics, invited Rasch to spend two months with the Cowles Commission on Economic Research at the University of Chicago, where he met L. J. Savage. This set the stage for Savage to bring Rasch back to Chicago in 1960, to finish writing Probabilistic Models and to give the series of lectures that introduced this writer to Rasch's new psychometrics.

Koopmans had also studied with Fisher in London, and Rasch had attended Koopmans' lectures while studying with Frisch in Norway in early 1935 (Bjerkholt, 2001, p. 9). Then, while in Chicago in 1947, Rasch collaborated with Koopmans (1975 Nobelist) and another of Frisch's students, a Norwegian Rockefeller Fellow, Olav Reiersol, in

> "a comparative study of problems arising from the specification of models, in particular problems of identification, in three fields: a multivariate model designed for the study of

> human metabolism; a model designed for the analysis of mental aptitudes; and an econometric model of the type employed by the Cowles Commission in its economic research. The similarities and differences encountered help to understand the nature of scientific induction."

This work, combined with Rasch's studies under Ronald Fisher in London 14 years earlier, would seem to have been vitally important preparation for the development of the unidimensional measurement models.

In sum, it appears that Rasch's presence in Chicago in 1947 was in large part due to his relationships with Frisch and Ronald Fisher, and his 1960 invitation from Savage stemmed from the acquaintance they formed when they were both working at the Cowles Commission and that they then cultivated through their participation in Biometric Society meetings. It was at one of these meetings, in 1959, that Rasch impressed Savage with his need to communicate his measurement work (Andrich, 1995, p. 1)

Having established the network of social connections leading to Rasch's presence in Chicago, what insight into the development of the intellectual substance of his work can we glean?

Frisch's Decisive and Indirect Intellectual Influence on Rasch

Rasch (in Andrich, 1979, 1997; Rasch, 1960, p. 21; Rasch, 1977; Wright, 1980) repeatedly and strongly emphasizes the vital importance of Ronald Fisher's concept of the sufficient statistic as an invaluable factor in the work informing his models, separability theorem, and epistemological focus on specific objectivity.

But further insight into Rasch's achievements, and a feeling for some of the issues at stake in economics at the time Rasch was with the Cowles Commission, emerges in a review of Frisch's work and influences (Arrow, 1960). Frisch was the editor of *Econometrica* for over 20 years, 1933-1954, and coined a lot of words we now take for granted, like "macroeconomics," “econometrics,” and others (http://cepa.newschool.edu/het/profiles/frisch.htm). Frisch was heavily influenced by the work of the Yale economist, Irving Fisher, one of the most important economists of the first half of the 20th century (Dimand, 1997: Dimand and Geankapolos, 2005; G. Fisher, 2005; Frisch, 1947). Frisch and I. Fisher were among the cofounders of both the Cowles Commission and the Econometric Society (of which I. Fisher was the first President).

One of I. Fisher’s primary areas of interest was in the mathematical structure of value and the indices used to measure it (I. Fisher, 1921, 1930, 1967/1922). Frisch (1930) was among the first and most effective to engage constructively with I. Fisher on these issues, basing his work in his own highly original axiomatic formulation of measurable utility (Frisch, 1926), which “introduced the axiomatic approach into the theory of economic choice” (Chipman, et al., 1971, p. 326; Boumans, 2001a, p. 333; Boumans, 2005). So, “while Fisher’s approach was to select among a number of index formulae by testing them, Frisch’s method was to derive mathematically the appropriate form of these tests” (Boumans, 2001a, p. 333). This aptly describes not only Frisch’s approach, but Rasch’s as well.

Irving Fisher could not, however, have worked from an infinite number of all possible index formulae, even if he was not as disciplined as Frisch in his selection of them. Insight into his principles is gained from consideration of his incurable tinkering; he was always working problems out in relation to some form of technical instrumentation or another. His most famous invention is now known as the rolodex. Significantly, one of his earliest papers was on Kant’s theory of geometrical axioms, and was entitled, “Mathematical Contribution to Philosophy.” This early involvement with geometry influenced I. Fisher’s use of mathematics in that he tended to define problems as exercises in instrument design (Boumans, 2001a, p. 316; 2005, p. 157).

His 1891 Yale PhD dissertation, “Mathematical Investigations in the Theory of Value and Prices,” written under the direction of J. W. Gibbs, was followed by his promotion to full professor at Yale less than seven years later. In the same year Rasch was with the Cowles Commission, Frisch celebrated I. Fisher’s 80th birthday in print, saying “…it will be hard to find any single work that has been more influential than Fisher's dissertation" (Frisch, 1947). Further, economists refer to the invariance of the real interest rate relative to equal and opposite changes in the expected inflation rate and the nominal interest rate as the “Fisher Effect” (for instance, see Koustas & Serletis, 1999), which is also referred to in terms of the “Fisher hypothesis” and the “Fisher equation.”

Fisher’s instrumental approach contrasts strongly with Frisch’s axiomatic approach. Fisher was concerned that the formulae from which indexes were derived should allow for “qualitative differences” that would be “a bridge to reality” (I. Fisher, 1967/1922, p. 274; quoted in Boumans, 2001a, p. 340). I. Fisher’s “rule that an instrument’s performance should approximate a standard within a satisfactory margin” (Boumans, 2001a, p. 341) gives a practical and applied perspective that can provide an important and often-needed contrast with overly pure mathematical approaches. Though no evidence of direct influence on Rasch from I. Fisher has yet come to light in this study, there is striking similarity between this practical orientation and Rasch’s (1960, pp. 37-8) sense that models are not meant to be true, but should be useful, and that they always remain on trial.

It is in this context that we might best appreciate the fact that Irving Fisher is widely known for what is called a separation theorem (I. Fisher, 1930, chapters 6-8). The basic principle is fundamentally the same as Rasch’s separability theorem, but with an economic twist. The theorem separates managerial opportunities for productivity from entrepreneurial market opportunities. The point is that a firm’s basic objective is the maximization of its current value, no matter what the investment preferences and financing sources of the owners happen to be.

The Fisher Separation Theorem posits that investment budgeting decisions are made in a two-stage process. First, entrepreneurial capital investment decisions are held to be independent of the preferences of the owner, and second, the investment decision is independent of the financing decision. The story told by these relations became the basis of neoclassical macroeconomic theory, and each of them could be written as a multifaceted Rasch model (Linacre, 1989),

$$\ln(P_{nij}/P_{nij-1}) = D_i - B_n - K_j$$

such that the log of the odds of observed investment probabilities P should be equal to the difference between pro and con decisions D as to investments i, owner n’s preferences B, and the source j of the financing K. The unidimensionality, additivity, and separability of each of these parameters is then independently evaluated, in the same manner as more conventional dichotomous or rating scale models.

The implications of I. Fisher’s separation theorem apparently prompted Frisch (1930) to formulate a paper on the necessary and sufficient conditions under which an index number shall satisfy the requirements of I. Fisher’s tests. Frisch does not cite R. Fisher (1922) on the sufficient statistic, but Rasch makes a point of saying that he “soon got hold” of Fisher’s paper after arriving in London in 1935 (Wright, 1980, p. xi; Olsen, 2003, p. 28; Wright and Olsen conflict on whether it was 1934 or 1935, but 1935 seems correct).

As is well known (Andersen, 1977; van der Linden, 1992), in Rasch’s models, counts of correct responses are what are today called minimally sufficient statistics. That is, these counts, or the analogous sums of ratings in the context of surveys or assessments, are minimally sufficient, and so, necessary, since they are functions of all the other statistics that are sufficient in the sense of summarizing data with no loss of information. What this means is that, “if there exists a minimal sufficient statistic for the individual parameter Theta which is independent of the item parameters, then the raw score is the minimal sufficient statistic and the model is the Rasch model” (Andersen 1977, p. 72). As Wright (1977b, p. 114) points out, it then follows that

> Unweighted scores are appropriate for person measurement if and only if what happens when a person responds to an item can be usefully approximated by a Rasch model.... Ironically, for anyone who claims skepticism about 'the assumptions' of the Rasch model, those who use unweighted scores are, however unwittingly, counting on the Rasch model

to see them through. Whether this is useful in practice is a question not for more theorizing, but for empirical study.

That is, if a count of correct answers or a sum of ratings can provide a meaningful basis for invariant, additive quantification, then a Rasch model holds. Even when data are not evaluated for fit to a Rasch model, even when the invariance and additivity properties of quantitative measurement are ignored, use of test, survey, or assessment scores as though they are measures inherently implies acceptance of Rasch's separability theorem.

This is because the parameter separation theorem is nothing less than a formal representation of the rigorous independence of figure and meaning, or of name and concept (Fisher, 2003a, 2003b, 2004b), that must be assumed in any communication, even in the discourses of deconstruction (Ricoeur, 1977, p. 293; Derrida, 1982, p. 229; Derrida, 1989, p. 218; Gasché, 1987, p. 5). Rasch's mathematics make tests of the qualitative hypothesis of quantitative meaningfulness (Narens, 2002) more accessible and practical than most work in this area. And in so doing, it taps deeply into the history of measurement and deploys rich possibilities for mathematical thinking that remain largely unexplored (Wright, 1988, 1997). Many of these possibilities reside in the field of econometrics (Fisher, 2002, 2004a, 2004c, 2008).

Frisch Between Fisher and Fisher, Relative to Rasch

As far as I've been able to ascertain, Rasch makes no mention at all of any influence of Frisch's work on himself apart from confluence analysis. Nor does he mention Frisch's close association with Irving Fisher, or I. Fisher's work. Instead, he repeatedly stresses the crucial importance of Ronald Fisher, to the point of having the reputation of being the "great communicator" of R. Fisher's ideas in Denmark (Olsen, 2003, p. 42). Rasch's student, Erling Andersen, remarked that he recalled Rasch quoting R. Fisher and using his examples well into the 1960s (Olsen, 2003, p. 28).

In contrast with this ongoing enthusiasm for Fisher's ideas, Rasch confessed to using Frisch's confluence analysis less as time passed because he did not "think there is anything in social sciences that is linear" (Olsen, 2003, p. 28). The latent influence Frisch exerted, however, may have been even more profound than the overt influence exerted by R. Fisher.

It often happens that the issues circulating at a time crystallize simultaneously in multiple works as independent developments. It is probably not just a coincidence that Rasch develops his separability theorem and bases it in statistical sufficiency in a context of proximity to Frisch, who thought through the conditions in which a minimally sufficient statistic would be possible for an economic index number, and whose close colleague I. Fisher had developed a separation theorem for the creation of rigorously constituted index numbers.

But the tantalizing question remains as to how, when, and why Frisch might have prepared Rasch for what R. Fisher had to teach him in London about sufficiency. Rasch was with Frisch only months before meeting Fisher, and Frisch had published his paper on minimally sufficient index statistics only five years before.

Could it be that Ronald Fisher's famously anti-economic rhetoric (Zilliak, 2007) insinuated itself into Rasch's perspective, to the point that he completely closed himself off from hearing or seeing anything related in Frisch's work? After all, where Irving Fisher and Frisch were the first president and a founding member, respectively, of the Econometric Society, Ronald Fisher and Rasch were, respectively, the first president and a founding member of the International Biometric Society.

The details of Rasch's relationship with Frisch that subtly prepared Rasch for what R. Fisher had to teach him in London about sufficiency may never be known. Many implications for the conduct of the social sciences, however, are hidden within the methodological overlaps existing between Rasch models and models commonly employed in econometrics, such as matched case-control conditional logistic regression models (Gautschi, 2001; McFadden, 1974; Rice, 2004). Kærgård (2003), for instance, suggests that Rasch's lack of impact on econometrics in his own day contrasts with the situation today in 1) the similarity between Rasch's concept of specific objectivity and today's emphasis on data mining methods (also see Linacre (2001) for an exploration of this theme), 2) the emergence of robust estimation methods less dependent on the normal distribution, which accords with Rasch's focus on distribution-free estimation (Kærgård, 1987), and 3) the established use of hazard functions and Weibull distributions involving the duration of waiting times, the length of periods of unemployment increases, or the durability of various goods (Kærgård, 1970; Kiefer, 1988).

These overlaps might have been expected had there been some awareness of, or way of recognizing, Rasch's implicit incorporation into his models of the principles first elucidated by I. Fisher and Frisch, principles now built into the foundations of econometrics. The conceptual identity of statistical sufficiency with mathematical invariance (Hall, Wijsman, & Ghosh, 1965) has led to a general consensus among statisticians "that statistical analysis should depend only on a sufficient statistic" (Arnold, 1988, p. 79; Arnold, 1985). Indeed, not only is the meaningfulness of quantitative scales contingent on demonstrable invariance (Falmagne & Narens, 1983; Narens, 1981, 2002; Mundy, 1986; Roberts, 1985, 1994), but *all* meaningful communication can be shown to require implicit qualitatively mathematical considerations of invariant reference (Fisher, 2003a, 2003b, 2004b).

It may turn out that the piecemeal and inefficient implementations of seemingly unrelated principles and methods introduced by the Fishers and Frisch are effectively and compactly integrated in the application of Rasch's models. Consider, for instance, Frisch's (1930, pp. 402-3) observation that

> The [previously referred to] formulae (8) and (9) must hold good with respect to any of the commodities entering into the index. In order that the index number Pts (with the continuity properties here assumed) shall fulfill the base test it is, therefore, necessary and sufficient that it be of the form
>
> Pts = Pt Qs (10)

where Pt is a function only of the prices p, and the quantities xt at the point of time t, and

Q, is a function only of the prices p, and the quantities x, at the point of time s.

This plainly builds out from I. Fisher's Separation Theorem and reads like a statement of a Rasch model. Could it be that the entire tradition of neoclassical economics is built on what are in effect Rasch models? It certainly appears so.

This conjecture is supported by recent conceptual analyses of economic modeling in which a distinction is made between descriptive and prescriptive measurement models (Boumans, 2001b). The generality of economic models depends more on being able to predict and explain facts about phenomena than on being able to account for variation in observed data. Boumans presents economic models in terms of instruments that could be calibrated so as to provide the invariant profiles needed for generality. He points out that "since its introduction to economics, calibration has been controversial," echoing long-standing debates and controversies in education, psychology, and health care concerning the epistemological status of Rasch's models (Wright, 1977a, 1984; Fisher, 1994; Andrich, 1988, 2002, 2004), and, more generally, concerning the difference between the statistical and measurement paradigms (Michell, 1986). Boumans (2001b, p. 429) points out that study results should not be dependent on the particular instrument or data-gathering method used, just as Rasch models have long been held to facilitate the isolation of invariances across respondent samples, instruments, time, and space.

Making no mention whatsoever of Rasch, Rasch models, Item Response Theory, or Latent Trait Theory, Boumans directly addresses the same issues of mathematical modeling, instrument calibration, reliability, invariance, and parameter estimation taken up routinely in these contexts. He (Boumans, 2001b, p. 430) also situates these issues in relation to metrological traceability to universally uniform reference standards, as has been pursued by Fisher (1997, 2000) relative to Rasch's models. He also contends that these issues are also of central concern in Haavelmo's (1944) highly influential probability approach in econometrics. Most important, perhaps, is the fact that Haavelmo's probability approach was highly influenced by Frisch and his method of confluence analysis, which is also highlighted by Bjerkholt (2001, p. 8).

Frisch's concern for the stability of evidence across the ephemeral vagaries of different sets of observations led him to the concept of autonomy as a matter of identifying stable economic relations (Boumans, 2001b, pp. 430, 440). Similarly, Rasch-based item banking, adaptive instrument administration, and judge/rater calibration methods (for more information, see chapters in Masters and Keeves, 1999; Fisher and Wright, 1994; Bond and Fox, 2007) are designed for situations in which the stability of the observational and inferential frames of reference is of paramount importance.

Frisch explicitly mentions a need for a bridge between his confluence analysis and R. Fisher's work. In an April, 1935, letter to Koopmans, Frisch refers directly to Ronald Fisher as having preceded Koopmans in the definition and solution of a problem in which the probability distribution of the second term in an observational series is not independent of the first term (Bjerkholt, 2001, p. 9). Both Haavelmo and Rasch attended lectures given by Koopmans at Frisch's invitation, in Norway in 1935 (Bjerkholt, 2001, p. 9), one of which involved R. Fisher's theory of estimation. Bjerkholt says that "an important part of Haavelmo's original inspiration towards the probability approach came from Frisch's ideas and from his work with Frisch on applying confluence analysis" (p. 8).

One wonders to what extent Rasch also might have found inspiration for *his* probability approach in his studies with Frisch concerning autonomy and confluence analysis, and in listening to Koopmans speak of Fisher's estimation theory. As Rasch later recounted, soon after arriving in London, he

> got hold of his [R. Fisher's] 1922 paper where he developed his theory of maximum likelihood, because I was especially interested in that matter. What caught my interest most was his idea that this is a form of generalization of just the same kind as Gauss attempted when he invented the method of least squares. The meaning of least squares is

> not, in Fisher's interpretation, just a minimization of a sum of squares. It is a maximization of the probability of the observations, choosing such values as estimates of the parameters as will maximize the probability of the set of observations at your disposal. ... This philosophy went further when Fisher got to his concept of sufficiency.....
>
> To purely mathematical minds sufficiency may appeal as nothing more than a surprising and singularly nice property, extremely handy when accessible, but, if not, then you just do without it. But to me sufficiency means much more than that. When a sufficient estimate exists, it extracts every bit of knowledge about a specified feature of the situation made available by the data as formalized by the chosen model. ‘Sufficient' stands for ‘exhaustive' as regards the feature in question....
>
> What is left over when a sufficient estimate has been extracted from the data is independent of the trait in question and may therefore be used for a control of the model that does not depend on how the actual estimates happen to reproduce the original data.
>
> The realization of the concept of sufficiency, I think, is a substantial contribution to the theory of knowledge and the high mark of what Fisher did.... His formalization of sufficiency nails down the ... conditions that a model must fulfill in order for it to yield an objective basis for inference.

Koopmans returned to London in November of 1935 to study further with R. Fisher, by which time Rasch was also there. To what extent did their conversations set the stage for Rasch to build a bridge between Fisher’s estimation theory and concept of sufficiency, on the one hand, and Frisch’s concept of autonomy and method of confluence analysis, on the other?

At about the same time, in October, 1935 (Bjerkholt, 2001, p. 10), Frisch wrote a letter to an American colleague explaining the need for a means of determining the extent to which the differences between his confluence analysis and R. Fisher’s sampling theory could be bridged. Frisch despairs in the letter of being able to find such a bridge in economics because of the difficulty of obtaining data that can be adequately controlled. Some years later, he advocated expanding the use of interviews with the aim of obtaining more control over the kinds of data gathered (Frisch, 1948, p. 370; in Boumans, 2001b, p. 447). That kind of control, and the kind of sampling from the universes of possible examinees and test items that would provide sufficient statistics, are precisely what Rasch focused on in the development of his probabilistic models.

Unfortunately, in contrast with Frisch, I. Fisher's place in this story has been obscured by accidents of history. He is remembered most notably "for his spectacular misprediction of stock prices in October 1929 and for eccentric crusades" even though he "emerges in retrospect as a major figure in the development of economics" (Dimand, 1997, p. 442; also see Cowles, 1967; Dimand and Geanakoplos, 2005; Fisher, 2005). Recent celebrations of Fisher's legacy have held that "Fisher is widely regarded as the greatest economist America has produced" and have elaborated on the large extent to which "much of standard neoclassical theory today is Fisherian in origin, style, spirit and substance" (Tobin, 2005, p. 19). And so it happens that

> Fisher vanished from citation lists by the 1940's as John Maynard Keynes captured the profession's attention, yet contemporary macroeconomics builds largely upon Fisher's foundations. … Fisher's contributions closely parallel much of modern macroeconomics, yet his role was long neglected. … The 'Mark I monetarism' of Friedman and his students had many Fisherian features …. Nonetheless, Friedman placed less emphasis on links with Fisher than with Chicago oral tradition (Dimand, 1997, pp. 442, 443).

But what issues and themes were likely at the heart of the Chicago oral tradition? Both Frisch and I. Fisher were instrumental in the establishment of the Cowles Commission (Christ, 1952), to the extent that the Chicago oral tradition was in many ways an extension of the conversation they started. The Cowles Foundation, as it is now called, itself holds that "The most important link [between it and I. Fisher] is intellectual. It is the great influence of Irving Fisher's economic thought in the entire range of topics of research activity at the Cowles Foundation" (Cowles, 1967).

Subtler links can also be discerned. Because of the death of Henry Schultz, the leading econometrician at the University of Chicago, in late 1938, “the University was in a position from which the possibility of adopting a group such as the Cowles Commission appeared particularly attractive. Likewise, the University was an ideal environment for the Cowles Commission” (Christ, 1952). The move of the Cowles Commission from Colorado in 1939 to Chicago thus took place in a context in which the Chicago oral tradition had been interrupted. Macroeconomics’ general Fisherian themes would have been even more pronounced in an econometrics organization that Fisher had helped launch, and in a context in which new leadership was imperative.

Chicago economics department faculty held positions with the Cowles Commission from 1939 on, and the role of the Cowles Commission in the intellectual vibrancy of the Chicago oral tradition is also evident in the number of future Nobel Prize winners who first came to Chicago as Cowles research associates, such as Koopmans, Arrow, and Simon.

When the Cowles Commission for Economic Research left the University of Chicago for a new home as the Cowles Foundation, it was no coincidence that it moved to Yale University, with which I. Fisher was associated for most of his career, 1890-1935. The 1967 Cowles Report of Research Activities states that, “the Cowles Foundation is now contributing to the continuation at Yale of the approach to economic theory and measurement so brilliantly initiated and represented by Irving Fisher.” One could hardly go far wrong in positing the continuation of I. Fisher’s approach to economic theory and measurement at Chicago, as well, in the field as a whole, and in the social sciences at large.

Rasch in Relation to Fisher, Frisch, and Fisher

It appears that Georg Rasch's Separability Theorem emerged as an implicit synthesis of Irving Fisher's Separation Theorem, Ragnar Frisch's proof concerning the necessary and sufficient conditions for satisfying I. Fisher's theorem, and Ronald Fisher's formulation of sufficiency. Three lessons can be drawn from these historical considerations.

First, Frisch may have capitalized on the opportunity created by Rasch's supervisors in Copenhagen, Nørlund and Madsen, when they sent Rasch to London to study with R. Fisher (Andrich, 1997, p. 542; Wright, 1980), by suggesting that R. Fisher's work on statistical sufficiency could be particularly important. Rasch spent time with Frisch in Olso for three months before joining Fisher in London, and Frisch's 1930 paper is strong evidence of the value he placed on the concept of sufficiency. Far from discovering sufficiency as a concept directly from contact with R. Fisher, Rasch may have arrived in London primed by Frisch to be looking for it.

Second, given these developments, we should be less surprised to find econometric models being applied to "everything from tax evasion to teenage pregnancy" (Hayes, 2007), since the mathematics used in these models is basically the same mathematics used in a wide variety of measurement models for test, survey, assessment, questionnaire and rating scale data across the social sciences.

Third, the connection between measurement as defined by Rasch and others (Wright, 1997) and the definition of capital as requiring additive, divisible, and transferable representations (De Soto, 2000; Fisher, W. 2002, 2008) now has a more fully elaborated context. Further research will be required to explore the similarities and differences between Irving Fisher's sense of capital and more recent work relating it to social networks and metrological infrastructure (Barzel, 1982; Latour, 1987, pp. 223, 247-57; NIST, 1996; Benham and Benham, 2000; De Soto, 2000).

Interestingly, I. Fisher and R. Fisher may have had an opportunity to meet under the auspices of the Cowles Commission, as they both spoke at a conference sponsored by the Cowles

Commission at its first home, in Colorado Springs, in the summer of 1936 (Christ, 1952), the year after Rasch had been in London studying with R. Fisher and in Olso with Frisch.

How might the history of economics, psychology, education, and social science been different if Rasch had been able to get the two Fishers and Frisch to focus on the importance of statistical sufficiency and maximum likelihood estimation in the context of blended instrumental and axiomatic approaches to tests of a separation theorem? Might we have today fewer meaningless but statistically significant results, and more in the way of substantively significant and quantitatively meaningful established scientific laws? One can only guess, and work to follow through on what's been started.

This odd coincidence—Rasch's relationship with Ronald Fisher and Rasch's formulation of a separability theorem from Fisher's concept of the sufficient statistic, alongside Frisch's relationship with Irving Fisher and Frisch's construal of necessary and sufficient statistics for Fisher's separation theorem—is made even more intriguing by the fact that Irving Fisher's biography at http://www.swlearning.com/quant/kohler/stat/biographical_sketches/bio20.1.html is accompanied by a photo of Ronald Fisher! (The same photo is identified correctly at http://www.asqstatdiv.org/pioneers.htm and many other web sites. It is an understandable error to make, given that both were white-haired men with glasses, mustaches and goatees. For a photo of Irving Fisher, go to the Cowles Foundation web site at http://cowles.econ.yale.edu/news/cowles/fisher_cf.htm or http://cowles.econ.yale.edu/conferences/fisher.htm.)

Concluding Notes

A commentator on econometrics recently made some telling remarks about the growing application of economic models in a growing number of areas of the social sciences, saying that

> the neoclassical model didn't leave its mark only on economics. In an audacious burst of methodological imperialism, Chicago Schoolers like Gary Becker used the framework of

> rational individuals seeking to maximize their utility to analyze and explain everything from tax evasion to teen pregnancy (Hayes, 2007).

What Hayes refers to as econometrics' methodological imperialism could arguably be seen as covering a lot more ground than the mere hegemony of neoclassical economics. For one thing, the alternative "heterodox" approaches emerging in economics that Hayes describes might be even more relevant to the analysis and explanation of everything from tax evasion to teen pregnancy than the neoclassical framework.

As Rasch implicitly understood, the basic structure of the of the relations built into the neoclassical model assumptions concerning parameter separation and statistical sufficiency can be observed to hold in any number of different situations. And as Rasch also implicitly understood, individual choice behavior is likely motivated as much or more by what people find meaningful and/or accessible than by the pure maximization of utility functions.

In retrospect, what Hayes refers to as methodological imperialism may come to be seen as something other than imperialism, at least other than a freely chosen or ideologically-motivated imperialism. What I have in mind (Fisher, 2003a, 2003b, 2004b) is more along the lines of what Thoreau (1854, p. 15) was referring to when he spoke of people being the tools of their tools. Nietzsche (1967, 466) got to the point, too, in his comments on the victory of scientific method over science. This insight into yet another source of factors impinging on our subjectivity comes as a new addition to a sequence of decenterings that humanity has periodically suffered or evolved through. Copernicus, Kepler, and Newton removed the earth from the center of the universe. Darwin removed humanity from the center of earthly creation, showing that we have evolved like any other form of life. Freud showed that we are not even the masters of our own consciousnesses, but instead play out the archetypes of the classical myths and legends that define roles and relationships.

We are today struggling to come to grips with yet another of these decenterings. This one might be considered a meta- or mega-decentering, insofar as it concerns the question of technology (Heidegger, 1977b; Ihde and Selinger, 2003; Harman, 2005). Technology's influence on life is not limited to the complexities of electronics, computers, the Internet, industry, or advanced weaponry. Rather, we must learn to manage a world by means of a stewardship that focuses on the creation and nurturing of living meaning, because technology is all-pervasive. We do not usually stop to think of them as such, but our clothing, buildings, roads, and furniture are all technological. Even the air we breathe and the food we eat are maintained with reference to metrological standards.

Giving ourselves over to this decentering is then not a matter of mere passive submission to some arbitrary standard, but connotes an acceptance of metaphysical limits. The constant pressure of the quest for greater efficiency and more general application that defines existence today is embodied in technology and is fundamentally mathematical. Mathematical thinking took on a life of its own at the time of its birth in ancient Greece, and it is now reaching a decisive developmental transition (Heidegger, 1977a). To take a prominent example of the mathematical project's maturational process, the emergence of global investment guidance based in metrics for human, social, and natural capital means that soon economists and financiers will be seeking to maximize the advantages that can be gained from solidifying the connections between Rasch's models for measurement and the foundations of econometrics laid down by Irving Fisher and Ragnar Frisch. With huge stakes at risk, these investment managers are highly pragmatic actors on the world stage who will not quibble over quaint issues such as methodological hegemony or method idolatry.

They will instead be very focused on using the human, social, and natural capital resources defined via precision measurement to maximize the value of their investments. By their very nature, living capital resources are organic and require careful, nurturing attention. One would hope that the universally uniform measurability of learning, development, growth, and of physical, psychological, social, community, and environmental health, etc. would lead to their better management, but that will not be the case always or everywhere. Attending to the critical needs of properly stewarding our human, social, and natural resources and the measures that represent them is a mission worthy of the researchers and academics in all fields.

Finally, it is important to note that the basic structure of the three-part relationships posited in economic models proposed by Irving Fisher and many others has been adopted in a wide range of other fields. As is pointed out by Rasch (1960, p. 115), this basic structure does not originate in economics, but in physics. Some (Ramsay, et al., 1975, p. 258) have speculated whether there is perhaps good reason why mathematical laws must take the particular form of three parameters, with one equal to the product or the quotient of the other two. Others (Burdick, Stone, and Stenner, 2006) have demonstrated the structural analogy that holds between the parameters for pressure, temperature, and volume that hold in the combined gas law, on the one hand, and the parameters for reader ability, text difficulty, and comprehension rate in a Rasch reading law, on the other. Restating the question Burdick and colleagues ask, how many such variable triplets structured by strict quantitative causal laws exist in the human, social, and environmental sciences? Whether recognized as such or not, the discovery of such laws will be by means of Rasch models, out of the mathematical necessity of its minimally sufficient statistics and out of the metaphysical necessity of the nature of existence.

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