Geometric and Algebraic Formulations of Scientific Laws:

Mathematical Principles for Phenomenology

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Key words: geometry; measurement; scientific law; Rasch models;

In review, *Journal of Phenomenological Psychology*
17 October 2011

Abstract

Geometry is the earliest practical science. The Pythagorean Theorem is among the first formalized scientific laws. Applying the natural logarithm to the terms in the Pythagorean Theorem gives the paradigmatic form of scientific laws in general. Identical results are produced across a series of equations, such that it is impossible to determine the domain from which any implied empirical relational structure has been drawn before being mapped onto a numerical relational structure. This lawful superstructure is extended into the social sciences in the context of a meta-theoretical framework for the discovery/invention of scientific laws. Practical implications include definition of a workable metatheory, projection of universally uniform units of measurement in an intangible assets metric system, a basis for systematic improvements in the efficiency of human, social, and natural capital markets, and the potential for a closer dialogue between phenomenological and mainstream approaches to psychological and other phenomena.

"Wise men, Callicles, say that the heavens and the earth, gods and men, are bound together by fellowship and friendship, and order and temperance and justice, and for this reason they call the sum of things the 'ordered' universe, my friend, not the world of disorder or riot. …you pay no attention to these things in spite of your wisdom, …unaware that geometrical equality is of great importance among gods and men alike, and you think we should practice overreaching others, for you neglect geometry."

Plato, *Gorgias*, 508

Geometry provides a model of scientific understanding that has repeatedly proven itself over the course of history. Einstein (1922) considered geometry to be "the most ancient branch of physics." He accorded "special importance" to his view that "all linear measurement in physics is practical geometry," "because without it I should have been unable to formulate the theory of relativity" (p. 14).

Burtt (1954) concurs with this sense of practical geometry as physics, pointing out that the essential question for Copernicus was not "Does the earth move?" but, rather, "...what motions should we attribute to the earth in order to obtain the simplest and most harmonious geometry of the heavens that will accord with the facts?" (p. 39). The Pythagoreans themselves considered the tonal proportions of musical scales to be the geometry of motion, encompassing not only sound but also celestial bodies and the human soul (Isacoff, 2001, p. 38). In landmark advances in the history of science, Maxwell employed a geometrical analogy in working out his electromagnetic theory, saying

> By referring everything to the purely geometrical idea of the motion of an imaginary fluid, I hope to attain generality and precision, and to avoid the dangers arising from a premature theory professing to explain the cause of the phenomena. If the results of mere speculation which I have collected are found to be of any use to experimental philosophers, in arranging and interpreting their results, they will have served their purpose, and a mature theory, in which physical facts will be physically explained, will be formed by those who by interrogating Nature herself can obtain the only true solution of the questions which the mathematical theory suggests (Maxwell, 1890/1965, p. 159).

Maxwell was known for thinking visually, having once as a student offered a concise geometrical solution to a problem that resisted a lecturer's lengthy algebraic efforts (Forfar, 2002, p. 8). His approach seemed to be one of playing with images with the aim of arriving at simple mathematical representations, instead of thinking linearly through a train of analysis. A similar method of mental imagery was used by Einstein (Holton, 1988, pp. 385-388).

The value found in geometry by Einstein, Maxwell, Newton, and many others is rooted in the earliest developments in the discipline, at the foundations of philosophy. Gadamer (1980) speaks of the mathematical transparency of geometric figures to convey Plato's reasons for requiring mathematical training of the students in his Academy, saying:

> Geometry requires figures which we draw, but its object is the circle itself.... Even he who has not yet seen all the metaphysical implications of the concept of pure thinking but only grasps something of mathematics—and as we know, Plato assumed that such was the case with his listeners—even he knows that in a manner of speaking one looks right through the drawn circle and keeps the pure thought of the circle in mind. (p. 101)

Plato required experience with geometrical figures because they "school one's vision for that which is thought purely" (Gadamer, 1980, p. 100). They accordingly then prepare students for philosophical study, standing as they do as models of "*all* those things which one can know through thought alone" (Gadamer, 1980, p. 101). Husserl (1931/1962) similarly held that

> If analogy can give any guidance at all in matters of method, its influence should be felt most strongly when we restrict ourselves to material mathematical disciplines such as geometry, and therefore ask in more specific terms whether a phenomenology must be built up, or can be built up, as a *'geometry' of experiences* (p. 185; original emphasis).

Husserl was then particularly interested in finding his way to a geometry of experience, to the point that

> The mathematical object seems to be the privileged example and most permanent thread guiding Husserl's reflection . . . [on phenomenology] because the mathematical object is ideal. Its being is thoroughly transparent and exhausted by its phenomenality (Derrida, 1989a, p. 27).

Accordingly, its "universality and objectivity make the ideal object into the 'absolute model for any object whatsoever'" (Bernet, 1989, p. 141, quoting Derrida, 1989a, p. 66). Many who have been attracted to phenomenological philosophy and methods for their qualitative meaningfulness and rigor may be surprised to encounter these emphases on the mathematical object (Hintikka, 2010). Though Husserl's first and last books were on arithmetic and geometry, respectively, and though Heidegger was skilled enough in mathematics to serve on his university's mathematics department dissertation committees (Krell, 1977, p. 12; also see Kisiel, 2002, p. x), few are aware of or seek out the development of mathematical themes in these writers' works (Tasić, 2001; Fisher, 2003a, 2003b, 2004, 2010a; Fisher & Stenner, 2011a).

The same is true of the works of others, such as Valéry, whose mathematical interests have yet to be brought into the foreground (Krauthausen, 2010), and Derrida (1981, pp. 34-35), who called for the liberation of the mathematization of language, claiming that

> The effective progress of mathematical notation goes along with the deconstruction of metaphysics, with the profound renewal of mathematics itself, and the concept of science for which mathematics has always been the model.

Insofar as phenomenology describes the processes by which things themselves are apprehended, any authentic, valid and productive science ought to imply sound phenomenological principles, no matter how obscured they may be by the epistemological preconceptions of the practicing scientist. As Ricoeur (1967, p. 219) observed, "A good implicit phenomenology is often concealed in the most objectivistic sciences." One might expect, then, that phenomenology would be replete with studies of mathematical objects as ideal models of a wide variety of constructs. Of particular value from this point of view would be studies documenting the rhythmic pattern of steps in the improvised dance of reduction and reflexivity that takes place as meaning is abstracted from the lifeworld (Finlay, 2008).

That expectation notwithstanding, Husserl's phenomenological studies and the ensuing deconstructions and interpretations of the history of metaphysics undertaken by his students, Heidegger (1962, 1967) and Gadamer (1980, 1989), in particular, as well as Derrida (1982, 1989a) and others, have not led to any mathematical laws, quantitative methods, or numerical analyses. Even direct considerations of the need for probabilistic approaches in phenomenology (Findlay, 1967) or of the relation of phenomenology to mathematics (Hartimo, 2010; Mancosu & Ryckman, 2002, 2005; Tieszen, 2005) give no practical guidance for researchers interested in applying phenomenologically-informed mathematical models or methods in the design of studies or the analysis of data. Nor has the extension of phenomenology into the philosophy of science and technology (Crease, 1993; Ginev, 1997; Glazebrook, 2000; Harman, 2005; Heelan, 1983, 1998; Ihde, 1991, 1998; Ihde & Selinger, 2003; Kockelmans & Kisiel, 1970) yet produced any works offering algebraic or geometric models of recognized phenomenological importance or general application in the social sciences.

Ironically, phenomenology and hermeneutics are generally viewed as postmodern perspectives allied with deconstruction and anti-scientific attitudes, instead of, more correctly, proceeding in parallel with developments in mathematics (Tasić, 2001). Significant contributions to understanding the dance of subject and object in science, following Gadamer's (1989, pp. 101-134) sense of play as the best clue to authentic method, are as often produced from non-phenomenological perspectives (Bohm, 1980; Capra, 2000; Zukav 2001) as from phenomenological (Crease, 1993). This is so despite the mathematical themes and concern with the hermeneutic construction of scientific objects sometimes developed at length (Harman, 2005; Heelan, 1987; Ihde, 1997). The basic principles of the phenomenological method and of the phenomenological reduction have been associated with the mathematics of Rasch's widely applied separability theorem (Fisher, 1992, 2003a, 2004, 2010a; Fisher & Stenner, 2011a), but, again, nothing explicitly formulated or characterized in terms of Husserl's goal of a geometry of experience was offered.

To begin to take steps in the direction of a closer dialogue between phenomenological and mainstream approaches to psychological and other phenomena, as called for by Giorgi (2009, p. 108), it is helpful to review some basic principles. For instance, exactly how are geometrical visualizations expressed algebraically? More specifically, how is it possible to see the algebraic structure of scientific laws in geometry? And what are the implications for models in the social sciences structured in the form of geometric regularities and scientific laws? How might the geometric imagery facilitating major advances in the history of science be approached in the psychosocial sciences? And finally, in what sense can the entities or constructs studied via a geometry of experience be considered real? Can a mathematical essentialism be avoided? Does a focus on the material practices of networked inscriptions propagated across media absolve such a geometry from such epistemological sins (Ginev, 2009)? Or might inscriptions be signs primarily of theoretical objects caught up in a temporal flow of ever-changing research, formally indicated in a covariant realism (Crease, 2009)? Or are these questions rendered irrelevant insofar as instruments capable of mediating relationships within an industry give rise to a new regime of soluble problems situated in an ongoing trajectory of technical improvements (Miller & O'Leary, 2007; Fisher & Stenner, 2011b)?

These questions must be answered in a way that remains accessible to those untrained in the interpretation of mathematical symbols. Thurstone (1959, p. 10) remarks on the need for students with the flexibility of mind needed for creative scientific work, saying that the visualization of essentially mathematical formulations is often more important than the capacity for mathematical calculation and symbolization. Of course, Thurstone concludes, "More fortunate is the student who has all these aptitudes."

In a similar vein, the economist Frisch (1947) recounts a story in which he had been critical of his friend and colleague, Irving Fisher, as the latter had produced a long verbal explanation of ideas that could have been much more concisely expressed in mathematical notation. Some years later, after discovering that attendance at his lectures was dwindling because the students could not follow his equations, Frisch encountered a student who happily reported having figured out what Frisch was saying by reading a paper of Fisher's. Frisch (1947,

p. 3) had no problem eating crow, as he says “You may imagine the joy I got out of writing to friend Fisher and telling him of this story."

In this writer’s own experience, colleagues have repeatedly found that qualitative elaborations of the implications of mathematical presentations—sometimes their own—provided them with insights not previously grasped. Perhaps the following combination of qualitative and quantitative explanations of elementary principles will work to inform the mathematically unskilled as to their own essentially mathematical formulations, and the mathematically erudite as to their own substantive implications.

This work is then undertaken in the spirit of Derrida’s (1981, p. 35) call for a slow and prudent extension of mathematical notation that proceeds along with, or, perhaps more aptly, begins to keep pace with, the deconstruction of metaphysics. After illustrating the equivalence of additive and multiplicative formulations of geometric and scientific laws, this structure is related to that of Rasch’s measurement models (Andrich, 1988, 2010; Bezruzcko, 2005; Bond & Fox, 2007; Rasch, 1960; Wilson, 2005; Wright, 1977; Wright & Stone, 1999), commonly employed in testing, survey, and assessment research and practice in education, psychology, and health care. Though the mathematically sophisticated will find the demonstrations provided exceedingly elementary, it is hoped that others who would otherwise be unable or unwilling to address themselves to material of this kind will come away from it with new appreciations for the power of qualitatively informed quantification. If there is any merit in the issues raised here, fuller and more complete mathematical treatments will no doubt soon be forthcoming from others interested in providing them.

The Geometry and Algebra of Scientific Laws

Starting from the Pythagorean theorem, we know that the square of a right triangle's hypotenuse is equal to the sum of the squares of the other two sides. For convenience, imagine that the lengths of the sides of the triangle, as shown in Figure 1, are 3, 4, and 5 units, for sides a, b, and c, respectively. We can count the unit squares within each side's overall square and see that the 25 in the square of the hypotenuse equals the sum of the 9 in the square of side a and the 16 in the square of side b. Employing a Cartesian coordinate system, that mathematical relationship can, of course, be written as

$$a^2 + b^2 = c^2$$

which, for Figure 1, is

$$3^2 + 4^2 = 5^2 = 9 + 16 = 25$$

Now, most scientific laws are not written in this additive form (which also includes equations involving subtraction), but in a multiplicative form (which also includes equations involving division), like this:

$$a = f / m$$

or

$$f = m * a$$

where the acceleration of an object can be estimated by dividing the applied force by the object's mass, and which, of course, is how Maxwell (1876/1920) presented Newton's Second Law.

This kind of equation is of the same form as many other natural laws (Crease, 2004). For instance, the Combined Gas Law's relations of volume, temperature, and pressure, or Ohm's law regarding voltage, current, and resistance, are both also structured in this way. In fact,

> …virtually all the laws of physics can be expressed numerically as multiplications or divisions of measurements. Although this rule has been known for a long time and forms the basis of the techniques of dimensional analysis widely used in engineering and physics, it remains a phenomenon for which no satisfactory explanation has been forthcoming (Ramsay, Bloxom, & Cramer, 1975, p. 258).

Scientific laws' multiplicative expressions may be rooted in geometry as practical physics, following Einstein. The formula for estimating the circumference of a circle by multiplying the radius squared by pi is of the same kind:

$$C = \pi * r^2$$

If this multiplicative structure is primarily or originally geometrical, how would the Pythagorean Theorem look, written multiplicatively, in the same form as a physical law?

To answer this question, we have to introduce the concept of the natural logarithm and the number *e* (2.71828...) (Maor, 1994). Since the advent of small, cheap electronic calculators, slide rules have fallen out of fashion. But these eminently useful tools are built to take advantage of the way the natural logarithm and its base in the number *e* make division interchangeable with subtraction, and multiplication interchangeable with addition.

These equivalencies make it possible to write the Pythagorean Theorem in the same form as Newton's Second Law of Motion, or the Combined Gas Law. The Pythagorean Theorem is normally written as

$$a^2 + b^2 = c^2$$

Using the convenient values for a, b, and c from above,

$$3^2 + 4^2 = 5^2$$

becomes

$$9 + 16 = 25 .$$

With this example in mind, it can plainly be seen that simply changing the plus sign to a multiplication sign will not work, since 9 * 16 is 144 and not 25.

This is where the number *e* comes in. What happens if *e* is taken as a base raised to the power of each of the parameters in the equation? Does this equation work?

$$e^9 * e^{16} = e^{25}$$

Substituting a for e^9, b for e^{16}, and c for e^{25}, this could be represented by

$$a * b = c$$

and could be solved as

$$8103 * 8{,}886{,}015 \approx 72{,}003{,}378{,}611$$

Yes, it works, and so it is possible to divide through by e^{16} and arrive at the form of the law used by Maxwell:

$$8103 \approx 72{,}003{,}378{,}611 / 8{,}886{,}015$$

or

$$e^9 = e^{25} / e^{16}$$

or, again substituting a for e^9, b for e^{16}, and c for e^{25}, could be represented by

$$a = c / b$$

and which, when converted back to the additive form using the natural logarithm, looks like this:

$$\ln(8{,}103) = \ln(72{,}003{,}378{,}611) - \ln(8{,}886{,}015)$$

and this

$$9 = 25 - 16 .$$

The Pythagorean Theorem provides the prototype for the structure of natural laws, then, in the sense that the empirical structure of physical relations is mapped into a theoretical structure of numerical relations in the same way that geometrical relations are.

A Geometry of Psychosocial Experience

Maxwell presented Newton’s Second Law in this form:

$$A_{vj} = F_j / M_v .$$

So when catapult j's force F of 7.389 Newtons (53.445 poundals) is applied to object v's mass M of 1.6487 kilograms (3.635 pounds), the acceleration of this interaction is 4.4817 meters (14.70 feet) per second, per second. (That is, 7.389 / 1.6487 = 4.4817, or 53.445 / 3.635 ≈ 14.70).

Increases in force relative to the same mass result in proportionate increases in acceleration, etc., just as increases in one variable result in proportionate changes in the other variables for other laws of the same form (Burdick, Stone, & Stenner, 2006). Furthermore, the empirical relational structure stays the same no matter what unit characterizes the numerical relational structure. In this context, Rasch (1960, 112-113) noted that,

> If for any two objects we find a certain ratio of their accelerations produced by one instrument, then the same ratio will be found for any other of the instruments. Or, in a slightly mathematized form: The accelerations are proportional.

Conversely, it is true that if for any two instruments we find a certain ratio of the accelerations produced for one object, then the same ratio will be found for any other objects.

Gadamer (1980) relates the fundamental idea of proportionality to Plato's original philosophical distinction between name and concept, pointing out that, for instance, the Pythagorean Theorem posits the independence of figures from the meaning they carry. Gadamer goes so far as to recognize that, "If I really know how to prove it [a theorem], I am no longer dependent upon the different possible figures or drawings which are used in the proof" (p. 103). He further observes that,

> Characteristic of a proportion is that its mathematical value is independent of the given factors in it, provided that they keep the same proportion to one another. The same relation can exist even when the numbers in it are changed. The universality of the relationship as such transcends its components. (p. 150)

What Gadamer is noticing here is the quality of mathematical transparency characteristic of numeric relationships, and of empirical relationships when they are meaningfully represented by numbers. These proportionate ratios constitute the abstract meaning structures referred to as the logos and are the metaphorical root of the concepts of logic and rationality (Gadamer, 1979, p. 4).

Perhaps a satisfactory explanation as to why virtually all the laws of physics are expressed as divisions or multiplications of measurements follows from the way the same relative proportion can exist even when the particular numbers involved are different. It has long been observed that, "In quoting quantitative empirical laws, scientists frequently neglect to specify the various scales entering in the equations" (Falmagne & Narens, 1983, p. 287). This phenomenon arose in the previously given example in which mass, force, and acceleration were shown in both the *Systeme Internationale* (SI) and American or British Imperial units. In stating the law, there is no need to indicate which unit is employed because the pattern of relationships is invariant across changes in scale, satisfying the criterion of meaningfulness (Mundy, 1986).

Of course, it is precisely here, in the constancy of natural laws' proportionate ratios across different combinations of numbers, where the connection with the variation hidden in the sameness is lost, and where the lived experience of the world is abstracted into the structures of meaning embodied in the mathematical languages of laws and defined units. Huge social and economic resources are poured into the disappearing act through which local practices and individualities are erased en route to enshrining universal laws and metric standards (Latour, 1987, p. 251; Schaffer, 1992, p. 42).

The convenience, utility, and elegance of the mathematical generality and rigor that are returned from these investments stand in marked contrast with the statistical results produced by the social sciences, where the specific characteristics of the scales used almost always must be known before valid inferences may be drawn. Roche (1998, p. 26) unintentionally identifies the conceptual route by which this fundamental difference between the natural and social sciences came to pass. Roche points out that, in the absence of unique and accurately maintained international units of measurement, quantities are “expressed as ratios and proportions whenever possible, since these are independent of the choice of unit.” But, and this is crucial, the meaningful invariance of those ratios and proportions across changes in scale requires that the units chosen must also be invariant. This was typically the case in the first scientific revolution, before the introduction of international metric standards. But a range of percentages based in counts of different sized units, as with correct test answers or survey ratings, are not even comparable with themselves, much less with percentages based in counts of entirely different sets of different sized units.

The social sciences remain immersed in individual differences and lack the universal uniform metrics and multiplicative laws of the natural sciences because of a failure to understand and act on the full meaning of the logos as a guide to authentic methods. That said, any viable framework for scientific psychosocial theory and practice will need to integrate into its methods recognition of the simultaneous disclosure of meaningful significance and concealment of its variable sources.

There is a tradition of research in the social sciences in which a far closer correspondence with the natural sciences is achieved, and in which the connection with individual idiosyncracy is not lost. In the manner of many scientists and philosophers before him (Black, 1962; Boumans, 2005, pp. 24-31; Fisher, 2010b; Heilbron, 1993; Myers, 1983, pp. 65-76; Nersessian, 2002, 2008; Turner, 1955), Rasch (1960) appropriated the proportionality modeled in Newton's Second Law. Unlike most others, Rasch's mathematical training led him (Rasch, 1960; also see his 1961, p. 325) to formulate a separability theorem in terms that apply to both additive and multiplicative forms of the models, saying

> It is possible to arrange the observational situation in such a way that from the responses of a number of persons to the set of tests or items in question we may derive two sets of quantities, the distributions of which depend only on the test or item parameters, and only on the personal parameters, respectively. Furthermore, the

conditional distribution of the whole set of data for given values of the two sets of quantities does not depend on any of the parameters. (p. 122)

Citing Maxwell's presentation of Newton's Second Law as the source for the mathematical form he sought (Rasch, 1960, pp. 110-115), Rasch (1961, p. 322) then wrote his model for measuring reading ability and text reading difficulty in the multiplicative form of

$$\varepsilon_{vi} = \theta_v \sigma_i$$

and also (Rasch, 1961, p. 333) in the additive form

$$\varepsilon_{vi} = \theta_{v} + \sigma_i \ .$$

These forms of the model assert that reading comprehension ε is the product (or the sum) of person v's reading ability θ and item i's text complexity σ. The model is also often written in the equivalent forms of

$$\Pr\{X_{ni} = 1\} = e^{\beta n - \delta i} / 1 + e^{\beta n - \delta i}$$

or

$$P_{ni} = \exp(B_n - D_i) / [1 + \exp(B_n - D_i)]$$

or

$$\ln[P_{ni} / (1-P_{ni})] = B_n - D_i$$

which all effectively say that the log-odds of a correct response from person *n* on item *i* is equal to the difference between the estimate B of person *n*'s ability and the estimate D of item *i*'s difficulty. Moving the effect of *e* from one side of the equation to the other makes the response odds equal to *e* taken to the power of the difference between B and D, divided by one plus *e* to that power.

So what happens if a couple of arbitrary values are plugged into these equations? If someone has a measure of 2 logits (log-odds units), what is the probability of a correct answer on an item that calibrates at 0.5 logits? (Relative to a uniform test centered at 0.0 logits, a person with a 2 logit measure is likely to score about 88 out of 100 questions correct. Conversely, given a sample of examinees for whom the test is appropriately targeted, an item in that test located at 0.5 logits is likely to be answered correctly by about 40% of the students.) The answer should be

$$\Pr\{X_{ni} = 1\} = e^{2-0.5} / (1 + e^{2-0.5}).$$

Now,

$$e^{2-0.5} = e^{1.5} = 2.71828^{1.5} = 4.481685....$$

and

$$4.481685 / (1 + 4.481685) \approx 0.8176$$

So the probability of success for person *n* of ability 2.0 logits on item *i* of difficulty 0.5 logits is about .82, with odds of .82 / .18, about 4.5 to 1. This is the same thing as saying

$$\ln[P_{ni} / (1-P_{ni})] = B_n - D_i$$

since

$$\ln[P_{ni} / (1-P_{ni})] = 2.0 - .0.5$$

and

$$\ln(0.82 / 0.18) = \ln(4.556) \approx 1.5 = 2.0 – 0.5 .$$

We can use the number *e* again to work the same example from Rasch’s multiplicative form of the model. Since subtraction corresponds with division and addition with multiplication we need to come up with the equivalent expression of

$$1.5 = 2.0 – 0.5 ,$$

which is

$$2.0 = 1.5 + 0.5 .$$

Now, using *e* we convert to a multiplicative form to get to Rasch's model

$$\varepsilon_{vi} = \theta_v \sigma_i$$

and have, with the previous values entered

$$e^{2.0} = e^{1.5} * e^{0.5} ,$$

which is

$$7.389 = 4.4817 * 1.6487 .$$

Dividing through by 1.6487 gives

$$4.4817 = 7.389 / 1.6487 .$$

Using the natural logarithm to convert division to subtraction, we get the same equation as above:

$$\ln(4.4817) = \ln(7.389) – \ln(1.6487)$$

which reduces to the same equation in use here:

$$1.5 = 2.0 – 0.5 .$$

Similarly, Newton's second law can be rewritten in an additive form, using the base *e* natural logarithm, like this:

$$A_{vj} = F_j - M_v .$$

Plugging in the values used in the example above, we get

$$\ln(4.4817) \approx \ln(7.389) - \ln(1.6487)$$

which reduces approximately to

$$1.5 = 2.0 - 0.5 .$$

Historical Context

In light of this exact identity in the mathematical form of his model for measuring reading ability and Newton's Second Law, Rasch (1960, p. 115) asserted that,

> Where this law can be applied it provides a principle of measurement on a ratio scale of both stimulus parameters and object parameters, the conceptual status of which is comparable to that of measuring mass and force. Thus, ... the reading accuracy of a child ... can be measured with the same kind of objectivity as we may tell its weight ....

Wright (1997, p. 44), a physicist who worked with Nobelists Townes and Mulliken before turning to psychology and collaborations with Rasch, concurs, saying, "Today there is no methodological reason why social science cannot become as stable, as reproducible, and hence as useful as physics." Andrich (1988, p. 22) observes that "...when the key features of a statistical model relevant to the analysis of social science data are the same as those of the laws of physics, then those features are difficult to ignore."

Rasch's appropriation of the form of Newton's Second Law via Maxwell for purposes of psychological measurement is rooted ultimately in Galileo, "who derived his rule relating time and distance using geometry" (Heilbron, 1998, p. 129). Pledge (1939, p. 144) makes the connection in the general point that

> as the Greeks gave us the abstract ideas (point, line, etc.) with which to think of space, and the 17th century those (mass, acceleration, etc.) with which to think of mechanics, so

> Carnot gave us those needed in thinking of heat engines. In each case the ideas are so pervasive that we use them even to state that they never apply exactly to visible objects.

Rasch worked in an academic environment infused with this idea (Fisher, 2010b). Heilbron (1993, pp. 5-6) documents how Newton's theory of gravitation provided the form of a Standard Model adopted across the sciences of nature in the late eighteenth century as the hallmark criterion of scientific success.

> Beginning around 1770…electricity, magnetism, and heat began to yield to the sort of analysis that had ordered the motions of the planets; and just after the turn of the 19th century, the phenomena of capillarity and the behavior of light were brought into the scheme…. These achievements inspired and exemplified the program described by Laplace in 1796 and brought almost to realization (or so he thought) by Gay-Lussac in 1809: to perfect terrestrial physics by the same techniques as Newton had used to perfect celestial mechanics.

The realization by Rasch that the Standard Model built on Newtonian mechanics could be extended from the domain of deterministic physical models to that of probabilistic psychosocial models is a remarkable accomplishment but remains inadequate to the task of creating mature sciences in education, health care, social services, etc. The psychosocial sciences have experienced a proliferation of analyses based in Rasch's models, but nothing close to the productivity of the natural sciences has yet resulted. This is because applications of Rasch's work to date in general lack several necessary features present in the natural sciences by about 1840 (Alder, 2002, pp. 328, 330; Daston, 1992, p. 338; Hacking, 1983, p. 234; Heilbron, 1993, p. 274; Kuhn, 1977, p. 220): the focus of theory on predictive control of the object of investigation, the focus on instrumentation embodying the substantive meaning of the numbers treated as measures, the focus on standard units interpretable at the point of use in a mathematical language shared by all members of a community of research and practice, and systematic education and training in research methods employing these conceptual tools. Little change in the productivity of the psychosocial sciences is likely until they are similarly empowered with theory, instruments, shared mathematical languages, and trained workforces.

Implications for Analytic Methods

Different methods of data analysis with different advantages and disadvantages are used to produce estimates of the modeled values, though they all converge on about the same values (Linacre, 1999). Furthermore, different models are available for different kinds of observations, such as those from ability tests, performance assessments, surveys, checklists, judge-mediated examinations, etc. (Wright & Mok, 2000). In the same way, different methods of varying strengths and weaknesses are employed in analyzing the consistency of the observations supposedly conforming to the structural invariance (Karabatsos, 2003; Smith, 1996; Smith & Plackner, 2009).

Estimates are typically expressed in some kind of log-odds unit, or logit, which is referred to as such because it is a natural logarithm of odds ratios. Odds ratios are produced by dividing probabilities, such as the percentage of a maximum score obtained, by their inverse (the difference between 1.0 and the probability). Untransformed logit estimates of Rasch model parameters taken straight from software output usually range between -3.0 or so and 3.0 or so. These estimates are usually arbitrarily centered on an item mean of 0.0, so that the average measure is interpreted relative to the center of the item scale.

Of course, these logits may be linearly transformed into any range convenient to the end user. As explained by Wright (1997), the interpretation of logits or their linear transformations is no more complicated than the interpretation of inches, hours, pounds, degrees Fahrenheit, volts, or their metric equivalents. Interpretation is facilitated by having meaningful substantive connections between the thing measured and the number line, and by having practical applications that routinely associate particular values with particular observations. Difficulties in interpreting logits are likely less due to any inherent complexity in their composition than to their being encountered in contexts lacking substantive practical and widely general associations.

Though the focus of interpretation is often too exclusively concerned with mathematical considerations, at the expense of substantive ones, it is nonetheless important to understand the relationships between probabilities, odds, and logits. Usually, when the overall probability of success for an individual examinee across all items taken is 0.50, the odds ratio is 1/1, and the logit measure is 0.0 (assuming the scale is centered at zero and all examinees have responded to all items). As other individuals' measures B become increasingly larger than the average item difficulty D, the probability increases asymptotically toward 1.0, the odds ratio ranges from 1.0000…/1 to $+\infty$/1, and the logits range from 0.000… to positive infinity. As the measures B become increasingly smaller than D, the probability shrinks asymptotically toward 0, the odds ratio ranges from .99999…/1 to .00000…./1, and the logits range from -0.000... to negative infinity.

The linearization of scores, percentages, probabilities and odds in logits occurs in part as a function of the removal of the artificial minimum and maximum limits imposed by observing particular behaviors or responses scored within a necessarily limited observational framework. Linearity is achieved in many sciences via applications of the natural logarithm in this manner (Maor, 1994). For tables and figures illustrating the relationships between counts of correct answers or rating scale scores with logit differences, odds, and probabilities, see Wright & Stone (1979, 1999), Wright (1977), Bond and Fox (2007), or others.

Consistent and invariant structural relationships can be hypothesized and tested in Rasch-based instrument calibration experiments (Andrich, 2011; Bunderson & Newby, 2009; Wright & Stone, 1979). Reasonable matches between expected and observed response probabilities are posited for various differences between ability, attitude, or performance measures B_n and the difficulty calibrations D_i of the items on the scale, between different measures relative to any given item, and between different calibrations relative to any given person. Of course, any number of linearly associated non-interacting parameters may be added, as long as they are included in an initial calibration design in which they are linked together in a common frame of reference (Linacre, 1989).

Model fit statistics, principal components analysis of the standardized residuals, statistical studies of differential item/person functioning, and graphical methods are all applied to the study of departures from the modeled expectations (Karabatsos, 2003; Smith, 1996; Smith & Plackner, 2009). The empirical evaluation of construct validity and unidimensionality in measurement will likely always be an art, as there is no single statistical formulation of data consistency able to specify every conceivable failure of invariance that might be important.

Practical Applications

Box (1979) is most often cited for the assertion that models are not supposed to be true, but useful. Rasch (1960, pp. 37-38; 1964, pp. 24, 2, 3; 1973/2011), however, repeatedly made the same assertion some years earlier than Box. If the value of mathematical laws and models is to be judged not by their truth but by their usefulness, what practical applications of the geometry of psychosocial experience are there? There is nothing so practical as a good theory, as Lewin

(1951, p. 169) put it, and, accordingly, the ultimate practicality emerges from defining a universally uniform unit of measurement capable of supporting a distributed network of collective cognition (Akkerman, van den Bossch, Admiraal, Gijselaers, Segers, et al., 2007; Hutchins, 1995; Magnus, 2007; Nersessian, 2006). The fullest actualizations to date of the potential value offered by such units are found in the domains of educational and psychological measurement. The philosophical context and a general overview of these means will aid in understanding an example from education.

Philosophical origins

Modern philosophy originated in ancient Greece in close association with mathematical thinking (Bochner, 1966, p. 66; Heidegger, 1967). One of the reasons why Plato restricted geometry to the compass and straightedge was to free constructions from dependencies on mechanical contrivances that enabled unjustified copying of geometric elements. Copying angles, arcs, or line segments was the only way some problems, such as the squaring of the circle or the doubling of the square, could be solved. Plato realized that the objects of geometry are not the physical figures themselves, which is how Pythagoreans treated them, but abstract ideals that stand apart from any given figure. This perspective not only solved the catastrophic Pythagorean problem concerning the immeasurable irrationality of the square root of 2 (Gadamer, 1979, p. 4), but also made the impossibility of drawing perfectly precise figures irrelevant.

For instance, if triangles themselves had to embody perfectly their Pythagorean definition, then the Pythagorean Theorem would always be false, since the sides and hypotenuse of any triangle can be measured to degrees of precision that will render the two sides of the equation unequal. Closer inspection of the triangle in Figure 1 might show that the three sides do not have lengths of 3, 4, and 5, but of 2.99949, 4.0000001, and 4.9998725. Squaring the first two of these numbers gives this equation:

$$8.9969402601 + 16.00000080000001 = 24.99694106010001$$

However, squaring 4.9998725 gives 24.99872501625625. Though the discrepancy is less than 0.002, it meant that errors in drawing and measuring figures would be unavoidable. Even if figures were copied as precisely as technology would allow, there would always be variations in skill levels and the threat of new, more precise, technologies. The practical value of geometry as giving results independent of the particular figures drawn would then be defeated. Instead, we would fit triangles, circles, lines, and squares to different models that allow some parameters to interact. The very concept of laws independent of the particulars of data, instrument, and observer would never have taken shape.

But in actual fact, concepts are not proved or disproved by individual instances of the things to which names refer. As Kockelmans (1970, pp. 53-54) points out, the conceptual origin of geometry is

> found in the awareness that each geometrical figure in principle at least can always be made more perfect… It is clear that by going from the practically realizable perfection to the horizon of imaginable perfection, limit forms began to delineate themselves—invariant and unreachable poles toward which all further perfection keeps pointing. It is the task of geometry to be interested in those ideal forms.

Gadamer (1980, pp. 33-34) expresses the general principle, saying, "that which constitutes being a horse could never be proved or disproved by a particular horse." In other words, the empirical consequences of a concept are not tests of that concept, but the opposite, the consequences are tested by the concept. Everything not referred to by the concept is excluded from consideration in the application of an abstract ideal model that tests the “imminent internal coherence of all that is intrinsic” to that concept (Gadamer, 1980, p. 34; also see Fisher, 2004; Linacre, 1996a). This issue characterizes the debates between measurement researchers who advocate fitting data to models and those who advocate fitting models to data (Andrich, 2002, 2004, 2011; Fisher, 1994).

The matter of central concern is the authenticity of method and the capacity to heed Husserl’s call to return to the things themselves (Husserl, 1911/1965, p. 108; Gadamer, 1994, p. 171). Gadamer (1989, pp. 463-464) contrasts Hegel's sense of invalid method as "external reflection" with authentic methods embodying the movement of the object of investigation itself. The action of the thing itself is rooted in what people say in conversations, letters, interviews, focus sessions, and journal entries, and in their actions and decisions. Researchers who pay attention to what people say and do may find regularities and patterns in their observations. Many of these patterns may not be obvious at first glance but can perhaps be isolated by

(1) abstracting common themes from the words and deeds of the research participants,
(2) incorporating those themes in survey questions or assessment items,
(3) deliberately structuring those questions/items so as to provoke responses likely to vary consistently from less to more,
(4) posing those questions to a sample of a larger group of people belonging to the same population as the initial qualitatively studied sample, and by
(5) testing the hypotheses that
    a. a self-organizing invariant consistency emerges from the survey/assessment data and retains its properties across subsamples, and

b. the quantitative results from the scaling process correspond directly with the qualitative results so that the measures give numeric expression to differences already apparent in what is said verbally.

This sequence of steps is central to the Rasch-oriented methodological recommendations of Wright, Stone, and Enos (2000) and Fisher (2006). This process of being captivated by and caught up in the self-representative play of the thing itself illustrates the meaning of the etymological root of the word “method” in *meta-odos* (following along on the path taken by the thing itself). What this process reveals—when it works—are “things as they show themselves before the work of abstraction and theorizing has carved out a language of fixed essences for them removed from human praxis, history and culture” (Heelan, 1994, p. 369). The process of abstraction and theorizing does not, however, carve out a language of fixed essences removed from human praxis, history and culture until the persistent invariance of the construct across samples, instruments, laboratories, and observers is codified in a universally uniform standard unit. Until that happens, and as long as measures are generated primarily from data analysis and not from theory and calibrated instrumentation, research remains captivated with playfully repeating the experience of the activity of the thing itself.

One example (Fisher, 2004) of a return to the things themselves is illustrated by showing that the same invariant structure emerges from two samples of data from the Knox Cube Test (Stone, 2002a; Wright & Stone, 1979). The root sense of authentic method is further explored by Solloway and Fisher (2007) in the context of a mindfulness study that incorporates all six of the steps listed above. One sees here the full meaning of the Platonic mission to “save the phenomena” by understanding the means of their production well enough to reproduce them at will.

Plato thus proposed modeling a line as an indivisible plane, a point as an indivisible line, a circle as a closed arc always equidistant from a single point, etc. (Cajori, 1985, p. 26; Ricoeur, 1965, p. 202). This practical independence of figure and meaning in geometry embodies the root philosophical distinction between name and concept, and is why Plato required all students entering his Academy to be trained in mathematics (Cajori, 1985, p. 26; Dilke, 1987, p. 19; Gadamer, 1980, pp. 100-101; Heidegger, 1967, p. 75).

Until the time of Plato, there was no automatic association between mathematics and numbers. Anything capable of serving as an object of reference, that could be taught and learned, was mathematical (Heidegger, 1967; Kisiel, 1973; Fisher, 2003a). The root meaning of mathematics for the ancient Greeks was simply learning (Heilbron, 1998, p. 8; Descartes, 1961, p. 17). But the association of mathematics with numbers and equations was so strong even by the time Aristotle, one generation after Plato, that the original, broader sense of it was lost. This loss became a more acute problem with the advent of modern science, provoking Husserl's (1970) effort to recover Galileo's "fateful omission" of the means by which the geometry of the experience of physical nature had been built up.

The deconstruction of the history of metaphysics converges with phenomenologically apt mathematical models in a way that points the way to a revitalized geometry of experience in general (Fisher, 2003b). The practical consequences of developing and following through on a new relationship between mathematics and philosophy lead to a focus on readable and inscribable technology and instrumentation (Heelan, 1983, 1998; Ihde, 1991, 1998). Instruments capable of mediating social and economic relationships on a broad scale are of particular interest due to their roles in making markets (Miller & O'Leary, 2007). The material practices in which inscriptions are propagated across changes in media (Latour, 1987, 2005) vary according to the specifics of local conditions, but do so in ways that nonetheless contribute to the overall mediation effected by the object represented.

How? The problem is one of knowing when local variation overwhelms the general invariance. Local variation can be a quantitative function of measurement error, or a qualitative function of construct validity and the internal consistency of the observations. Rasch measurement separates these issues and clarifies them in ways not addressed in statistical assessments of reliability (Wright & Stone, 1999). Reliable precision in measurement is not obtained by accident, but should be designed into instruments by ensuring so far as possible that the questions asked (1) share a common, coherent focus; (2) vary markedly in the responses they are likely to provoke, and (3) are numerous enough to estimate locations with confidence. Instruments designed in adherence with these principles are likely to provide measures that are as qualitatively meaningful as they are quantitatively precise.

Practical theory: Defining a unit

The importance of geometrical proofs as demonstrations of valid understanding is practical in the sense of enabling control over phenomena on the basis of predictive models. For instance, if two angles of a triangle are known, the value of the third may be inferred. If pressure and volume are known, temperature may be inferred. The capacity to compel understanding by means of geometric proofs enabled previously unknown degrees of agreement among people. It was these degrees of agreement that allowed for new applications of the rule of law in the mapping and surveying of the world, and in the development of standard weights and measures (Alder, 2002).

Geometric proofs are a formal way of demonstrating understanding in the sense of putting something in your own words. If you can construct and justify your own figures so that they satisfy the axioms of geometry, all must agree that you know what you are talking about. And so arises Plato's question: why is it that we cannot compel understanding in human, social, and moral domains the way that we do in the natural domain? Given the longstanding availability of methods that effectively implement a geometry of behavioral, cognitive, and moral constructs (for instance, Dawson, 2002, 2004; Embretson, 1998; Green & Kluever, 1992), it must be recognized that at least part of the reason why may reside in reluctance to accept responsibility for the consequences of asserting that such a capacity to compel understanding may be viable. And though investigations of this possible capacity must be undertaken prudently and mindfully, it must also be recognized that applications of it are already underway in many areas of contemporary life and in many areas of the world.

And so, briefly, how is a geometrically constructed quantitative unit of measurement defined and realized? Measurement is often defined as an estimate of the ratio of a magnitude of a quantitative attribute to another magnitude accepted by convention as a unit (Cooper & Humphry, 2010; Humphry, 2011; Michell, 1997). The existence of the unit, then, cannot precede the existence of the magnitude identified as supporting division into ratios. As is extensively documented in the history of science (Heilbron, 1993; Roche, 1998; Kuhn, 1977, p. 213), lawful regularities are identified and studied qualitatively, often for decades or centuries, before quantification is possible. Kuhn (1977, p. 219) suggests, then, that the path from scientific law to measurement can rarely be traversed in the reverse direction.

What does this mean in practical terms in the psychosocial sciences? In the context of Rasch's models for measurement, it means that law-like patterns in the empirical relational structures of data from test or survey questions, ordinal observations, and response likelihoods are repeatedly exhibited across different sets of questions designed to measure the same thing, and across different samples of examinees or respondents. Similar questions fall in similar and often highly correlated orders and relative positions on the metric across instruments and samples, as do similarly highly correlated types of examinees or respondents (Dawson, 2002, 2004; Fisher, 1997). Confidence in the stability of such patterns emerges first when they are observed to hold across hundreds and thousands of analyses of thousands of questions, and of tens and hundreds of thousands, and even millions, of examinees and respondents (Bond, 2008; Masters, 2007; Rentz & Bashaw, 1977; Stenner, Burdick, Sanford, & Burdick, 2006). Additional confidence accrues as theories of the constructs measured prove their predictive validity and items posing particular difficulties can be written to the needed specifications (Dawson, 2002, 2004; Embretson, 1998; Embretson & Daniel, 2008; Green & Kluever, 1992; Stenner, et al., 2006; Stenner & Smith, 1982; Stenner, Stone, & Burdick, 2011; Stone, 2002a).

But the replicability of various consistently reproduced orders and relative positions across data sets is only the first (ontological) phase in the process of defining and deploying a standard (ontic) unit (Fisher, 2000, 2005, 2009; Latour, 1987, 2005; Wise, 1995). Before the emergence of widely accepted standard units of measurement in the early nineteenth century, magnitudes of length, weight, time, temperature, and electrical current were measured in locally variable units that confused commerce and science (Alder, 2002; Ashworth, 2004; Roche, 1998; Zupko, 1977). Today, similar confusion reigns in education, health care, and other areas in which universally uniform units of measurement could reduce confusion and support the harmonization of consumer choices and quality improvement efforts. The process will be akin to tuning the instruments of the psychosocial sciences to equal-tempered scales, so that all unit differences are held constant across transformations, just as musical notes now are across key changes (Stone, 2002b).

A good deal of research has succeeded in identifying quantitative magnitudes of a number of important constructs, but little effort has yet been invested in arriving at consensus agreement on the conventions of unit size and nomenclature necessary for fully integrating mathematics and measurement in the psychosocial sciences. Though virtually everything remains to be done in defining universally uniform units (Humphry, 2011), the viability of such reference standards is supported by the convergence of construct definitions across independent studies (Dawson, 2002, 2004; Fisher, 1997, 2009), and by repeatedly demonstrated predictive theoretical control over such constructs (Dawson, 2002, 2004; Embretson & Daniel, 2008; Stenner, et al., 2006).

Practical example: A standard metric for reading

In recent educational research and practice, theory, data, and instruments interact to inform individually customized lessons targeting the range of difficulty in the curriculum where students are most likely to be optimally challenged and engaged (Black, Wilson, & Yao, 2011; Griffin, 2007). Empirical test item difficulty orders and measures inform instruction by indicating where students are at relative to the progression of learning the material taught. The predictability of response probabilities, given a student's measure and an item difficulty, means that tests also can be targeted at each individual student, making testing both more informative and more efficient. Further, instruction and assessment are increasingly integrated in a single online process, so that studying a lesson is an interactive experience that documents new learning while it is taking place (Solomon, 2002).

For instance, the Rasch Reading Law (Burdick, et al., 2006) is structured so that the probability of a correct response, or the reading comprehension rate, will be about 75% when the reader's measure has the same numeric value as an item's difficulty, estimated as a function of text complexity. The average estimated scale value for a full-length article or book parallels the average estimated scale values for test items generated from that article or book.

When a reader with a reading ability of 600L encounters text at a 600L reading complexity, random test items generated from that text typically result in 75% correct responses. As reader ability goes up relative to a fixed text complexity, so does the comprehension rate and the expected percentage of correct answers. Conversely, as text complexity increases relative to a fixed reader ability, comprehension decreases at a predictable rate. A recent study of thousands of students had an average expected comprehension rate of 75% for 719 articles averaging 1150 words (Stenner, Burdick, Sanford, & Burdick, 2011). Total reading time was 9,794 hours and the total number of unique machine-generated comprehension items was 1,349,608. The theory-based expectation was 74.53% correct and the observed percent correct was 74.27.

The regularity of this pattern allows its expression in a uniform unit of reading measurement (Stenner, et al., 2006). For years, children's book publishers have made text complexity measures of all their titles widely available. Results from all major reading tests, and many other kinds of high stakes graduation and admissions tests, are routinely available in this unit. Many reading textbook publishers structure their curricula around weekly quizzes linked with online individualized instructional modules. Paraphrasing Heelan (1994, p. 369), universally uniform units of measurement like this are languages of fixed essences carved out from human praxis, history and culture by the work of abstraction and theorizing. In contrast with the emergence of mathematical thinking in the natural sciences, abstraction and theorizing in the context of the psychosocial sciences has not resulted in a complete disconnection from human praxis, history and culture. It may be that Rasch-based reproductions of length, distance, weight, and density measures from ordinal observations (documented in Fisher, 2009) signify important steps toward the recovery of what Galileo so fatefully omitted from his account of the origins of modern science.

Conclusion

The additive expression of the Pythagorean theorem, the multiplicative expression of natural laws, and the additive and multiplicative forms of Rasch models all participate in the same simultaneous, conjoint relation of two parameters mediated by a third. For those who think geometrically, perhaps the connections drawn here will be helpful in visualizing the design of experiments testing hypotheses of converging yet separable parameters. For those who think algebraically, perhaps the structure of lawful regularity in question and answer processes will be helpful in focusing attention on how to proceed step by step from one definite idea to another, in the manner so well demonstrated by Maxwell (Forfar, 2002, p. 8). Either way, the geometrical and/or algebraic figures and symbols ought to work together to provide a transparent view on the abstract mathematical relationships that stand independent from whatever local particulars are used as the medium of their representation.

Plato defined a line as an indivisible plane, a point as an indivisible line, and so on (Cajori, 1985, p. 26; Ricoeur, 1965, p. 202), reconceiving the elements of geometry in a way that rendered harmless the otherwise devastating consequences for Pythagoreanism of irrational line segment lengths. These redefinitions were the geometric equivalents of his root philosophical distinction between name and concept, and of the arithmetical distinction between numbers and particular things counted. Students admitted to Plato's Academy were required to have prior experience in geometry as an introduction to this essential contrast between the concretely real and the abstractly ideal (Gadamer, 1980, pp. 100-101; Heidegger, 1967, pp. 75-76).

Following in the wake of Copernicus' and Kepler's geometry of the heavens, Galileo's "ambiguous genius" was to simultaneously reveal the world as applied mathematics and cover it over again as a work of consciousness (Ricoeur, 1967, p. 163; Husserl, 1970a, pp. 23-59; Burtt, 1954, p. 204). This "fateful omission" of the means by which the world came to be understood mathematically imposes the need to reconstruct the idealizing operations that extract the abstract forms from the life world (Ricoeur, 1967, p. 162). Reactivating the meanings and motivations of geometric ideals, with an awareness of their mercurial capacity to conceal as much as they reveal, is vitally important to cultural re-orientation and renewal (Ruin, 2011).

It is then of paramount importance that mathematical thinking be recognized as not primarily concerned with numbers and equations, but with the projection of idealized expectations organizing experience. Number does not delimit the pure ideal concept of amount, but vice versa. The Newtonian laws project the uniformity of a universe behaving with lawful regularity, and so arose the necessity of the narrow sense of universally uniform measures (Heidegger, 1967, pp. 89, 91, 93). As laws analogous with the Newtonian laws emerge in the psychosocial sciences, a profound change in the notation and mathematical formalism of those sciences will also emerge, just as those changes also took place in the natural sciences in the wake of Newton's successes there (Roche, 1998, p. 145).

Counterbalancing the full union of mathematics and measurement in the human and social sciences will be acute awareness of the fact that ideas, such as mathematical/geometrical theorems, natural laws, or the structure of Rasch models, do not exist and are unobservable. Rasch (1960, pp. 37-38, 1973/2011) repeatedly stresses this, much as Heidegger (1967, p. 89) does relative to Newton's first law, which

> ...speaks of a body...which is left to itself. Where do we find it? There is no such body. There is also no experiment which could ever bring such a body to direct perception. But modern science, in contrast to the mere dialectical poetic conception of medieval Scholasticism and science, is supposed to be based upon experience. Instead, it has such a law at its apex. This law speaks of a thing that does not exist. It demands a fundamental representation of things which contradict the ordinary.

Butterfield (1957, pp. 17, 25-26, 96-98) concurs with this assessment of the way the law of inertia

> required a different kind of thinking-cap, a transposition in the mind of the scientist himself; for we do not actually see ordinary objects continuing their rectilinear motion in that kind of empty space….we do not in real life have perfectly spherical balls moving on perfectly smooth horizontal planes…without resistance and without gravity.

Butterfield (pp. 25-26) further asserts that "nothing could have been more important [in encouraging this habit of mind] than the growing tendency to geometrise or mathematize a problem." That is, the seemingly unrealistic ideals of unobservable conditions that made problems manageable and amenable to mathematical treatment provided the means by which objectification and abstraction from the lifeworld took place.

Science in this sense is not primarily descriptive and evidence-based; rather, it is prescriptive in the form of the data and the quality of the instrumentation needed to be able to make objective inferences. With measurement taken to a sufficient degree of precision, no actual triangle ever fits the Pythagorean theorem, no balls roll on frictionless planes, and there are no test, survey, or assessment results completely unaffected by the particular questions asked and persons answering. The value of abstract ideal mathematical models lies not in their truth, but in their usefulness (Rasch, 1960, pp. 37-38). The question is one of whether systematic implementation of such a model, for instance in the development and deployment of a new metric system for the constructs of the psychosocial sciences (Fisher, 2009), would solve otherwise insoluble problems, and would not do so at the expense of creating new classes of yet larger insoluble problems.

And usefulness cannot be conceived solely in terms of a single data set but must be considered systematically from the perspective of the implied law as the basis of a universally uniform language of comparison. Despite the fact that no actual geometric figures ever satisfy the mathematical ideal, the geodetic survey nonetheless serves the purpose of defining property rights. Even though no mercury thermometer ever embodies a perfect relation of temperature, pressure, and volume, this does not destroy the value of the device for baking a cake or deciding what to wear. Even though no length of insulated electrical cable ever perfectly exhibits the resistance properties expected by Ohm's Law, there is nonetheless an astounding array of consumer electronics products. Similarly, no written text or reading student will ever completely satisfy the Rasch Reading Law (Burdick, et al., 2006), but that law may nonetheless form a valid and useful basis for new electronic products across an array of industries.

Rasch extends Plato's philosophical distinction between name and concept, or between geometrical/arithmetical figure and meaning, into the psychosocial sciences. In the same way that a triangle is defined abstractly as mathematical relations never observed in practice, and in the same way that Newton's laws project idealizations not subject to experimental proof, so, too, does Rasch posit a relation of question and answer in which responses are dominated only by the difference between the ability, attitude, or performance of the person measured and the difficulty or challenge of the item or task used to measure.

Communicating the clarity and transparency of an idea requires careful attention to the objective representation of the relevant class of things observed. "The first concern of all dialogical and dialectical inquiry is a *care for the unity and sameness* of the thing under discussion" (Gadamer, 1991, p. 61). Even a deconstructive focus on unclear and obscure texts presumes care for the unity and sameness of an object of discourse (Derrida, 1982, p. 229; 1989b, p. 218; 2003, p. 62; Ricoeur, 1977, p. 293). If it did not, there would be nothing to write about.

Decades of successful application of Rasch's models (Bezruczko, 2005; Fisher & Wright, 1994) show that, so far as possible, the observational framework must be constrained by theory to produce observations likely to conform reasonably to the idea. When this is achieved, comparable measures of individuals are produced independent of the specific questions asked, meaning that it becomes possible to look right through the particulars of the local situation and keep the pure thought of the infinite population of all possible questions in mind. The practical value of this is that the measuring system remains open to the addition of new, or removal of old, construct-relevant questions—without compromising the comparability of the measures made. This capacity is the basis for adaptive instrument administration and item banking (Kisala & Tulsky, 2010; Lunz, Bergstrom, & Gershon, 1994), and theory-based on-the-fly item generation (Bejar, Lawless, Morley, Wagner, Bennett, et al., 2003; Embretson & Daniel, 2008; Stenner & Stone, 2003).

But reasonable conformity is not perfect, and observations that contradict the general regularity are the primary means through which new phenomena are discovered. Kuhn (1977, p. 205), like Rasch (1960, p. 124), recognized that one of the primary reasons for measuring is to reveal anomalies. This, indeed, is

> the systematic problem of philosophy itself: that the part of lived reality that can enter into the concept is always a flattened version—like every projection of a living bodily existence onto a surface. The gain in unambiguous comprehensibility and repeatable certainty is matched by a loss in stimulating multiplicity of meaning (Gadamer, 1991, p. 7).

In other words, "all interpretation makes its object univocal and, by providing access to it, necessarily also obstructs access to it" (Gadamer, 1991, p. 8). What Husserl (1970) named Galileo's "fateful omission" of the origins of his geometry of the experience of nature was a fundamentally hermeneutic event, in the sense of Hermes as the messenger god who steals and conceals at the same time he gives and reveals. Though Derrida primarily focused on the ambiguities and losses experienced in interpretation, he (1982, p. 229) similarly held the primary focus of philosophy, its "sole thesis," to be the rigorous independence of meaning from the figures of metaphor that permeate language. He (Derrida, 2003, pp. 62-63) accordingly emphasized the fact that "When I take liberties, it's always by measuring the distance from the standards I know." As Ruin (2011, p. 80) points out, Derrida never "solved" or abandoned his early inquiry into the reality and nature of ideality and meaning. The often opposing perspectives Gadamer and Derrida took from this common sense of philosophy led to Risser's (1989) image of the two faces of Socrates, the Gadamerian midwife comforting the afflicted and the Derridean gadfly afflicting the comfortable.

Thus the challenge of achieving unambiguous comprehensibility goes hand in hand with the danger of losing contact with individual uniqueness (Ballard, 1978, p. 189). Transparent meaning ironically emerges via an irreducible dialectic of productive revealing and a preceding constitutive always-already-thereness (Ruin, 2011). Reduction of the infinite potentialities of experience to discourses of limited length may commit the violence of the premature conclusion (Ricoeur, 1974), and so become reductionism, when representations are not justified as sufficient and necessary, and when once-justified representations are not vigilantly monitored. Qualitative reductionism is as much a danger as quantitative reductionism (Fisher, 2010a; Solloway & Fisher, 2007). Quantitative data and methods are no more inherently reductionistic than qualitative data and methods are. The issue in either case is how well argument and evidence are brought together to justify what is said.

Ideally, data quality requirements should be both necessary and sufficient for justifying inferences from a reduction of a potentially infinite array of possible questions and answers to a particular collection of individual questions and answers. This ideal is exactly what is posited in the mathematics of Rasch models: “If there exists a minimal sufficient statistic [i.e., one that is both necessary and sufficient] for the individual parameter Theta which is independent of the item parameters, then the raw score is the minimal sufficient statistic and the model is the Rasch model" (Andersen, 1977, p. 72; 1999; also see Andrich, 2010; Fischer, 1981; van der Linden, 1992). Probabilistic models of individual-level constructs provide a context within which reductions based in minimal sufficient statistics may be experimentally tested (Andrich, 2011; Bunderson & Newby, 2009). These models do not just accommodate error and data imperfections, but are actually stronger models than those based in purely deterministic structures (Engelhard, 1994; Linacre, 1996b; Wilson, 1989). The multiple approaches to the evaluation of data quality devised over the last 50 years and more (Smith, 1996; Smith & Plackner, 2009), then, are not afterthoughts or supplementary additions to the overall methodological framework, but are integral to that framework (Andrich, 2010; Bond & Fox, 2007; Wright & Stone, 1979, 1999).

Systematic attention to all three moments of the phenomenological method (reduction, application, and deconstruction) (Heidegger, 1982, pp. 19-23, 320-330; Fisher, 2010a; Fisher & Stenner, 2011a) is required of a science capable of living up to the demands of a geometry of experience. This is not as simple as it might sound. Rasch (1960, pp. 110-115; Fisher, 2010b) explicitly formulated his model of reading ability in analogy to Maxwell's model of Newton's Second Law, following Maxwell's own self-described method of analogy (Black, 1962; Nersessian, 2002; Turner, 1955). But, as is hinted by Maxwell (Larmor, 1937, pp. 17-18; Nersessian, 2002, p. 143) when he contrasts his method of analogy with Kelvin's, the cognitive operations involved in successfully employing the method are far more than mere analytical rules that can easily be followed and applied (Nersessian, 2008). Where Kelvin merely substituted parameters in one domain with those from another, Maxwell understood the need to allow the parameters within each domain to interact in their own characteristic way. Though expectations may be guided and testable hypotheses suggested by the analogy from another, better understood, system, the domain being investigated must be constructed in its own terms.

To apply a Rasch model is to accept the challenge of employing Maxwell's method of analogy, but what Rasch actually did was far closer to Kelvin's method than Maxwell's. The consequences of this for mathematical modeling in the psychosocial sciences have been extensive. Rasch models are widely applied as analytic tools with no cognizance of the relation of those models to the structure of natural law or to the need for substantive theories of the constructs measured. As awareness of the larger potentials grows (Andrich, 2002, 2004, 2011; Bond & Fox, 2007; Wilson, 2005), this situation will be rectified. Qualitative methods, in general, and phenomenological methods, in particular, are already becoming increasingly integrated with quantitative methods that focus on defining and identifying invariantly additive units of measurement (Dawson, Fischer, & Stein, 2006; Fisher, 2004, 2010a; Fisher & Stenner, 2011a). What Rasch's models do is guide the creation, application, and critique of inscribable media for the self-representative play of the things themselves (Fisher, 2004, 2010a; Fisher & Stenner, 2011a). More widespread understanding of what this means will enable communities of research and practice to find their collective voices, to see individual exceptions as proving the rules (in the sense of testing them), and to reveal human nature by means of those exceptions. In the end, what else could we have expected?

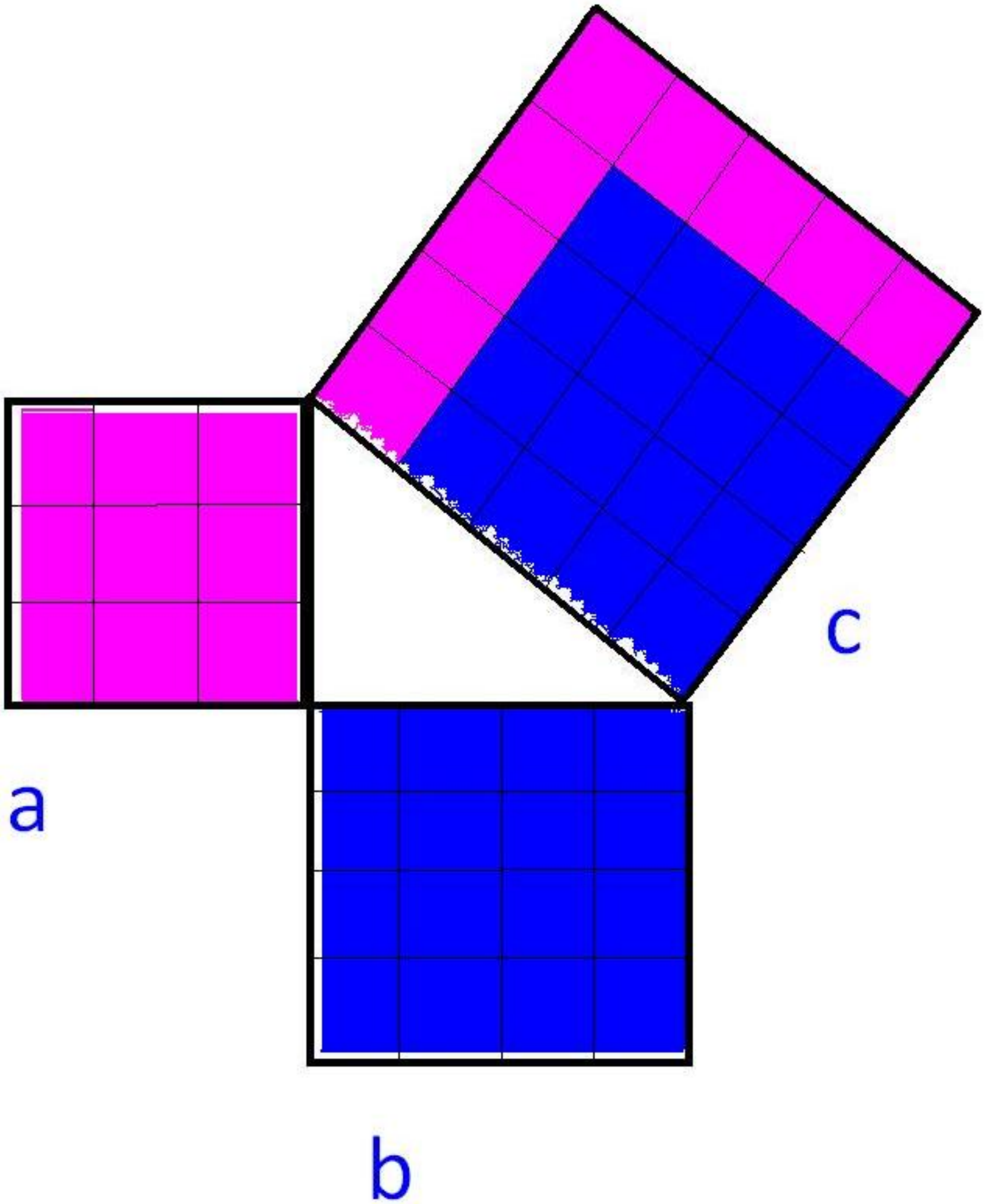

Figure 1. A geometrical proof of the Pythagorean Theorem

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